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A MODEL FOR THE CHARMED D_s^+ MESON
DECAYS INTO THREE PIONS

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Abstract

We propose a phenomenological three component model describing the decay amplitude of the process $D_s^+ \rightarrow 3\pi$. The first component is a constant background, the second one is a Breit-Wigner amplitude associated to a quasi two-body $f_0(980)\pi^+$ state and the third one another Breit-Wigner amplitude corresponding to a possible quasi two-body $\rho(770)\pi$ state. We show that it is possible to reproduce the observed total rate for $D_s^+ \rightarrow \pi^+\pi^+\pi^-$ as well as the two other measured branching ratios for the non resonant part and the resonant $f_0\pi^+$ one. Implication of the $\rho\pi^+$ state, of which an experimental limit has been given, is discussed. An upper bound in the 10 MeV range for the decay constant $f_{\pi'}$ of the $\pi(1300)$ meson is obtained .

Predictions are given for the $D_s^+ \rightarrow \pi^0\pi^0\pi^+$ rate as well as for the π^+ and π^- (π^0 and π^+) energy distributions for these two decay modes $D_s^+ \rightarrow \pi^+\pi^+\pi^-$ ($D_s^+ \rightarrow \pi^0\pi^0\pi^+$) respectively.

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I Introduction

We are interested, in this paper, in the decay of the D_s^+ meson into three pions. Experimental data are available only for the final state $\pi^+\pi^+\pi^-$ [1, 2], but not for the $\pi^0\pi^0\pi^+$ one. By inspection of the results, we observe that the quasi two body state $f_0(980)\pi^+$ has been clearly identified [1] whereas for the quasi two body $\rho^0(770)\pi^+$ state, no significant signal has been found [1]. Another remarkable feature of the data is an important fraction of the decay proceeds into a non resonant $\pi^+\pi^+\pi^-$ state [1], this fact necessarily implies a sizable W^+ annihilation contribution. More precisely, the rate for D_s^+ decays into a non resonant $\pi^+\pi^+\pi^-$ state is about 1/3 of the dominant spectator decay rate $D_s^+ \rightarrow \phi\pi^+$, both being governed by the same Cabibbo-Kobayashi-Maskawa (CKM) favored factor $V_{cs}^*V_{ud}$.

After a brief discussion of the kinematics for three body decay and a presentation of the experimental results in section II, we construct a three component model for the transition amplitude of the mode $D_s^+ \rightarrow \pi^+\pi^+\pi^-$. The first component corresponds to a quasi two body $\rho^0\pi^+$ state, followed by $\rho^0 \rightarrow \pi^+\pi^-$. Since ρ^0 meson is a $(u\bar{u} - d\bar{d})/\sqrt{2}$ isovector state, the decay $D_s^+ \rightarrow \rho^0\pi^+$ can proceed only through the W annihilation mechanism (henceforth WA). In section III, we propose a model for the $W^+ \rightarrow \rho^0\pi^+$ transition involving an intermediate virtual $\pi(1300)$ meson. The experimental upper limit for the branching ratio $\mathcal{B}(D_s^+ \rightarrow \rho^0\pi^+)$ allows us to derive an upper bound for the decay constant $f_{\pi'}$ of the $\pi(1300)$ meson to be around 10 MeV.

The second component is associated to the observed quasi two body $f_0\pi^+$ state followed by the decay $f_0 \rightarrow \pi^+\pi^-$. The quark content of the $f_0(980)$ meson is crucial for constructing the amplitude $D_s^+ \rightarrow f_0\pi^+$. If the f_0 is a pure $(u\bar{u} + d\bar{d})/\sqrt{2}$ isoscalar state, we must again use the WA, as for the ρ^0 previously. In section IV we show that by comparing the $D_s^+ \rightarrow \rho^0\pi^+$ and $D_s^+ \rightarrow f_0\pi^+$ rates, such an assignment $(u\bar{u} + d\bar{d})/\sqrt{2}$ for $f_0(980)$ is in strong disagreement with experiment. Then the $f_0(980)$ must contain a large $s\bar{s}$ component or even be a pure $s\bar{s}$ state. Now the spectator mechanism can be used for the decay $D_s^+ \rightarrow f_0\pi^+$, and from experiment, we estimate in section IV the magnitude of the hadronic form factor $F_0^{D_s f_0}(m_\pi^2)$ describing the $D_s \rightarrow f_0 W^+$ transition. If $f_0(980)$ is a pure $s\bar{s}$ state, we obtain, from experiment $F_0^{D_s f_0}(m_\pi^2) \approx 0.36$, a reasonable value.

For the third component associated to a non resonant $\pi^+\pi^+\pi^-$ state, we introduce a complex constant amplitude which corresponds to an uniformly populated Dalitz plot. Such a component is determined, from experiment, in section V where, in addition, the total rate for $D_s^+ \rightarrow \pi^+\pi^+\pi^-$ is computed using the three component model described above. The result, consistent with experimental data, is presented as a function of a possible phase for the constant non resonant amplitude.

The π^+ and π^- energy distributions for the decay $D_s^+ \rightarrow \pi^+\pi^+\pi^-$ are computed in section VI. The $f_0\pi^+$ quasi two body state is clearly seen in the π^+ energy distribution, as already obtained in [1], no $\rho^0\pi^+$ enhancement is present, and the π^- energy distribution does not reveal any particular structure even if it is very different from the pure phase space distribution. It is then straightforward to make predictions for the decay mode $D_s^+ \rightarrow \pi^0\pi^0\pi^+$ using isospin considerations relating the two final states $\pi^+\pi^+\pi^-$ and $\pi^0\pi^0\pi^+$. In section VII, we present our results for the rate and for the π^0 , π^+ energy distributions of the D_s^+ decay into $\pi^0\pi^0\pi^+$.

Finally, we end this introduction by few considerations concerning the quark content of the $f_0(980)$ scalar meson based firstly on SU(3) flavor symmetry and secondly on the decay products of the $f_0(980)$. The SU(3) flavor nonet $J^{PC} = 0^{++}$ contains an isovector meson $a_0(980)$, two isoscalar mesons $f_0(980)$ and $f_0(1300)$ and two isodoublets of strange mesons $K_0^*(1490)$. The physical particles $f_0(980)$ and $f_0(1300)$ are expected, as usual, to be the result of a configuration mixing between the isoscalar octet and singlet weights. We remark that the quasi degeneracy in mass between the $a_0(980)$ and $f_0(980)$ is reminiscent of that between the $\rho(770)$ and $\omega(782)$ in the $J^{PC} = 1^{--}$ nonet. A first naive expectation is that $f_0(980)$, like $\omega(782)$, is dominantly a $(u\bar{u} + d\bar{d})/\sqrt{2}$ bound state, the $f_0(1300)$ like $\phi(1020)$ being essentially a $s\bar{s}$ state. However such an assignment is wrong for at least two reasons from both theoretical and experimental considerations. Indeed if one tries to apply for the scalar $J^{PC} = 0^{++}$ nonet the usual mixing formalism which has been very successful in the $J^{PC} = 1^{--}, 0^{-+}, 2^{++}, 3^{--}$ cases, then we are faced with the impossibility to determine the mixing angle θ_s for scalar mesons. Because of the large mass of the $K_0^*(1490)$ meson, the formalism leads to $\tan^2\theta_s < 0$, no matter is used the quadratic or linear Gell-Mann - Okubo mass formula. Therefore the previous analogy between the $J^{PC} = 1^{--}$ and 0^{++} nonets is meaningless. The second reason is an examination of the experimental decay products of the $f_0(980)$ and $f_0(1300)$, in particular the $\pi\pi$ and $K\bar{K}$ channels. In spite of the tiny phase space available for $f_0(980) \rightarrow K\bar{K}$, its branching ratio 22% is actually a very large number. The situation is inverted for the second meson $f_0(1300)$, the available phase space for $K\bar{K}$ is much larger, however its branching ratio is only 7.5%, with 93.6% into $\pi\pi$. From these experimental considerations, we conclude that if the $J^{PC} = 0^{++}$ nonet is made of quark-antiquark bound states, the $f_0(980)$ must contain a large $s\bar{s}$ component and the $f_0(1300)$ a small $s\bar{s}$ one, with a $(u\bar{u} + d\bar{d})/\sqrt{2}$ component being dominant for $f_0(1300)$ and secondary for $f_0(980)$. Moreover, we shall see in section IV that the comparison of the $D_s \rightarrow \rho^0\pi^+$ and $D_s \rightarrow f_0(980)\pi^+$ rates also confirms the dominant $s\bar{s}$ component of the $f_0(980)$ meson. Such $s\bar{s}$ classification for $f_0(980)$ has also been recently advocated in [3].

II Generalities and Experimental results.

1). We study the decay of the D_s^+ meson of energy momentum P into three π mesons of energy momenta p_1, p_2, p_3 with the relation $P = p_1 + p_2 + p_3$. We introduce the Mandelstam variables s_1, s_2, s_3 and the π meson energies in the D_s^+ rest frame E_1, E_2, E_3 . At the level of accuracy of the experiments, it is legitimate to neglect the mass difference between charged and neutral pions and we get

$$\begin{aligned} s_1 &= (p_2 + p_3)^2 = (P - p_1)^2 = m_{D_s}^2 + m_\pi^2 - 2m_{D_s}E_1 \\ s_2 &= (p_3 + p_1)^2 = (P - p_2)^2 = m_{D_s}^2 + m_\pi^2 - 2m_{D_s}E_2 \\ s_3 &= (p_1 + p_2)^2 = (P - p_3)^2 = m_{D_s}^2 + m_\pi^2 - 2m_{D_s}E_3 \end{aligned} \quad (1)$$

Energy momentum conservation implies the relations

$$s_1 + s_2 + s_3 = m_{D_s}^2 + 3m_\pi^2, \quad E_1 + E_2 + E_3 = m_{D_s} \quad (2)$$

The double differential distribution is given in terms of the transition matrix element by the expression

$$d\Gamma = \frac{1}{64\pi^3} \frac{1}{m_{D_s}} | < 3\pi | T | D_s^+ > |^2 dE_1 dE_2 \quad (3)$$

We remind that the transition matrix element $< 3\pi | T | D_s^+ >$ involving three body final state is dimensionless. In the (E_1, E_2) plane, the phase space is defined by the constraints

$$\begin{aligned} m_\pi &\leq E_1 \leq \frac{(m_{D_s}^2 - 3m_\pi^2)}{2m_{D_s}} \\ E_-(E_1) &\leq E_2 \leq E_+(E_1) \end{aligned} \quad (4)$$

with

$$E_\pm(E) = \frac{1}{2} (m_{D_s} - E) \pm \frac{1}{2} \sqrt{\frac{(E^2 - m_\pi^2)(m_{D_s}^2 - 3m_\pi^2 - 2m_{D_s}E)}{m_{D_s}^2 + m_\pi^2 - 2m_{D_s}E}} \quad (5)$$

Of course, the mass difference between charged and neutral pions being neglected, we have the same phase space in the two other planes (E_2, E_3) and (E_1, E_3) . Two possible three pions final states are $\pi^+\pi^+\pi^-$ and $\pi^0\pi^0\pi^+$. We make the following choice of π meson variables

$$\begin{aligned} (1). \quad &\text{final state } \pi^+\pi^+\pi^-, & E_1(\pi^+), E_2(\pi^+), E_3(\pi^-) \\ (2). \quad &\text{final state } \pi^0\pi^0\pi^+, & E_1(\pi^0), E_2(\pi^0), E_3(\pi^+) \end{aligned}$$

2). Let us now discuss the experimental situation concerning the only observed 3 pion final state $\pi^+\pi^+\pi^-$. The decay rate for the mode $D_s^+ \rightarrow \pi^+\pi^+\pi^-$ has been first measured by the E 691

collaboration at Fermi Lab. [1] and the result has been confirmed few years later within error by the NA 82 collaboration at CERN [2]. In addition, the E691 group has been able to exhibit in the $\pi^+\pi^+\pi^-$ Dalitz plot a structure due to the $f_0(980)\pi^+$ quasi two body state. However a structure due to the $\rho^0(770)\pi^+$ quasi two body state has not been clearly identified. The various branching ratios in [1] are normalized to the most easily measured mode $D_s^+ \rightarrow \phi\pi^+$. Using the Particle Data Group value [4]

$$\mathcal{B}(D_s^+ \rightarrow \phi + \pi^+) = (3.5 \pm 0.4)\% \quad (6)$$

the branching ratios obtained in [1] are :

$$\mathcal{B}(D_s^+ \rightarrow \pi^+\pi^+\pi^-)_{TOT} = (1.54 \pm 0.42)\% \quad (7)$$

$$\mathcal{B}(D_s^+ \rightarrow \pi^+\pi^+\pi^-)_{NR} = (1.015 \pm 0.352)\% \quad (8)$$

$$\mathcal{B}(D_s^+ \rightarrow f_0\pi^+) = (0.98 \pm 0.38)\% \quad (9)$$

$$\mathcal{B}(D_s^+ \rightarrow \rho^0\pi^+) < 0.28\% \quad \text{at } 90 \% \text{ } C.L. \quad (10)$$

where, in (8) the subscript NR means non resonant [1]. Our results (8), (9), and (10) agree with those given in [4]. However the branching ratio of Eq.(7) quoted in [4] is slightly smaller, $(1.35 \pm 0.31)\%$, because an average has been made between the data obtained in [1] and [2]. For a question of consistency we shall use, in this paper, the set of data (7) – (10) in order to preserve the relative fractions of the various terms as quoted in [1].

III. The decay mode for $D_s^+ \rightarrow \rho^0 \pi^+ \rightarrow \pi^+ \pi^+ \pi^-$

1). Let us consider the quasi two body final state $\rho^0 \pi$. The transition matrix element has the form

$$\langle \rho^0 \pi^+ | T | D_s^+ \rangle = g_{D_s \rho \pi} (P_{D_s} + p_\pi)^\mu e_\mu^*(p_\rho) \quad (11)$$

where $e_\mu(p_\rho)$ describes the ρ meson polarization. The dimensionless quantity $g_{D_s \rho \pi}$ is a weak complex coupling constant in terms of which, the decay width is given by

$$\Gamma(D_s^+ \rightarrow \rho^0 \pi^+) = \frac{1}{2\pi} \frac{K_\rho^3}{m_\rho^2} |g_{D_s \rho \pi}|^2. \quad (12)$$

Here K_ρ is the center of mass momentum of the final $\rho\pi$ particles in the D_s^+ meson rest frame.

2). The ρ^0 and pion are quark-antiquark bound states containing only u and d quarks and the decay $D_s^+ \rightarrow \rho^0 \pi^+$ can only proceed through the WA, since both charm and strange quarks being absent in the final state. For the transition $W^+ \rightarrow \rho^0 \pi^+$ we assume, by the Partial Conservation of the Axial Current (PCAC) the contribution of an intermediate state having the quantum number of a π^+ meson [5, 6, 7] and which might be the π meson itself or its recurrence $\pi(1300)$. It has been checked that the π meson intermediate state gives a contribution many order of magnitude smaller than that of the $\pi(1300)$ and only the latter one is retained in the diagram of Fig.1.

Here $\pi' \equiv \pi(1300)$ and the corresponding expression of the weak decay constant $g_{D_s \rho \pi}$ is given by

$$g_{D_s \rho \pi} = a_1 \frac{G_F m_{D_s}^2}{\sqrt{2}} V_{cs}^* V_{ud} \frac{f_{D_s} f_{\pi'}}{m_{\pi'}^2 - m_{D_s}^2 - i m_{\pi'} \Gamma_{\pi'}(m_{D_s}^2)} g_{\pi' \rho \pi} \quad (13)$$

where a_1 is the phenomenological parameter introduced by Bauer, Stech, Wirbel (henceforth BSW) [8]; V_{cs}, V_{ud} the relevant CKM matrix elements; f_{D_s} and $f_{\pi'}$ are the leptonic decay constants of the D_s^+ and π' mesons respectively.

The dimensionless strong decay constant $g_{\pi' \rho \pi}$ is related to the corresponding decay rate by

$$\Gamma(\pi' \rightarrow \rho^0 \pi^+) = \frac{1}{2\pi} \frac{K_{\rho'}^3}{m_\rho^2} |g_{\pi' \rho \pi}|^2. \quad (14)$$

where $K_{\rho'}$ is the C. M. momentum in the π' rest frame of the $\rho\pi$ final state.

Combining Eqs. (12), (13) and (14) we obtain

$$\mathcal{B}(D_s^+ \rightarrow \rho^0 \pi^+) = \frac{\tau_{D_s}}{\hbar} \left(\frac{K_\rho}{K_{\rho'}} \right)^3 f_{\pi'}^2 \Lambda^2 \mathcal{B}(\pi' \rightarrow \rho^0 \pi^+) \quad (15)$$

where Λ^2 is given by

$$\Lambda^2 = a_1^2 \left[\frac{G_F m_{D_s}^2}{\sqrt{2}} \right]^2 |V_{cs}|^2 |V_{ud}|^2 f_{D_s}^2 \frac{\Gamma_{\pi'}(m_{\pi'}^2)}{(m_{\pi'}^2 - m_{D_s}^2)^2 + m_{\pi'}^2 \Gamma_{\pi'}^2(m_{D_s}^2)} \quad (16)$$

3). For masses, widths and CKM matrix elements we use the values collected by the Particle Data Group [4]. The quantity f_{D_s} is chosen to be $f_{D_s} = 280 \text{ MeV}$ consistent with recent experimental data [4] and theoretical expectations. For the decay $\pi(1300) \rightarrow \rho\pi$ we use [4]

$$\mathcal{B}(\pi' \rightarrow \rho\pi) = 0.3205 \quad (17)$$

and we leave $f_{\pi'}$ as a free parameter. For a_1 we take $a_1 = 1.26$ [9].

In the following, we use the simple energy dependence $m\Gamma(s) = \sqrt{s} \Gamma(m^2)$ for the widths, although more sophisticated expressions have been proposed [5, 6]. Because of the not too large difference between the π' and D_s^+ masses, we expect the sensitivity of Eqs.(13) and (16) to different forms of $\Gamma_{\pi'}(m_{D_s}^2)$ to be relatively modest.

We now write the branching ratio $\mathcal{B}(D_s^+ \rightarrow \rho^0 \pi^+)$ in the form

$$\mathcal{B}(D_s^+ \rightarrow \rho^0 \pi^+) = C^2 f_{\pi'}^2 \quad (18)$$

We retain only in the numerical computation of C^2 the large uncertainty due to the $\pi(1300)$ width [4], $\Gamma_{\pi'} = (400 \pm 200) \text{ MeV}$ and we get

$$C^2 = 16.3 \pm_{7.4}^{5.1} \text{ GeV}^{-2} \quad (19)$$

Using now the 90% confidence level upper limit $\mathcal{B}(D_s \rightarrow \rho^0 \pi^+) < 0.0028$ of Eq.(10) we deduce an upper limit for $f_{\pi'}$

$$|f_{\pi'}| < \begin{vmatrix} 11.45 \\ 13.11 \\ 17.72 \end{vmatrix} \text{ MeV} \quad \text{for} \quad \Gamma_{\pi'} = \begin{vmatrix} 600 \\ 400 \\ 200 \end{vmatrix} \text{ MeV} \quad (20)$$

Theoretical value for $f_{\pi'}$, as large as 40 MeV , seems to be overestimated in the literature [7]. Eq.(20) gives, to our knowledge, the first experimental information on $f_{\pi'}$, and our result is compatible with theoretical calculations using non-relativistic chiral quark model [10] or QCD sum rule technique [11].

4). We now take into account the non-zero ρ meson width effect. The production of the quasi two body state $\rho^0 \pi^+$ is followed by the decay $\rho^0 \rightarrow \pi^+ \pi^-$. For the final state $\pi^+ \pi^- \pi^+$, E_1 and E_2 are the π^+ energies and E_3 is the π^- energy. The decay amplitude for $D_s^+ \rightarrow \rho^0 \pi^+ \rightarrow \pi^+ \pi^- \pi^+$ is given by

$$\langle \pi^+ \pi^- \pi^+ | T | D_s^+ \rangle_{\rho\pi} = g_{D_s \rho \pi} g_{\rho \pi \pi} \left\{ \frac{s_2 - s_3}{m_\rho^2 - s_1 - i m_\rho \Gamma_\rho(s_1)} + \frac{s_1 - s_3}{m_\rho^2 - s_2 - i m_\rho \Gamma_\rho(s_2)} \right\} \quad (21)$$

and the corresponding decay rate has the form

$$\Gamma(D_s^+ \rightarrow \rho^0 \pi^+ \rightarrow \pi^+ \pi^- \pi^+) = \frac{1}{64 \pi^3} m_{D_s} |g_{D_s \rho \pi}|^2 |g_{\rho \pi \pi}|^2 \frac{J}{2} \quad (22)$$

where J is the phase space integral

$$J = \frac{1}{m_{D_s}^2} \int \int dE_1 dE_2 \left| \frac{s_2 - s_3}{m_\rho^2 - s_1 - i\sqrt{s_1}\Gamma_\rho} + \frac{s_1 - s_3}{m_\rho^2 - s_2 - i\sqrt{s_2}\Gamma_\rho} \right|^2 \quad (23)$$

The energy dependence of the ρ meson width has been taken as explained previously and the factor $\frac{1}{2}$ in Eq.(22) is due to the presence of two identical π^+ in the final state.

The dimensionless constant $g_{\rho\pi\pi}$ is related to the ρ meson width by

$$\Gamma(\rho^0 \rightarrow \pi^+\pi^-) = \frac{1}{6\pi} \frac{K_\pi^3}{m_\rho^2} (g_{\rho\pi\pi})^2 \quad (24)$$

where K_π is the C. M. momentum in the ρ^0 meson rest frame of the two pions.

Combining Eq.(22) and (24) we obtain

$$\Gamma(D_s^+ \rightarrow \rho^0\pi^+ \rightarrow \pi^+\pi^-\pi^+) = \frac{3}{8\pi^2} \frac{m_{D_s}}{m_\rho} |g_{D_s\rho\pi}|^2 \frac{\Gamma(\rho^0 \rightarrow \pi^+\pi^-)}{(1 - 4m_\pi^2/m_\rho^2)^{3/2}} J \quad (25)$$

It is straightforward to estimate the importance of the non-zero ρ^0 meson width. The result is obviously independent of $g_{D_s\rho\pi}$ and comparing Eq.(12) and (22) we get

$$\frac{\Gamma(D_s^+ \rightarrow \rho^0\pi^+ \rightarrow \pi^+\pi^-\pi^+)}{\Gamma(D_s^+ \rightarrow \rho^0\pi^+)} = \frac{3}{4\pi} \frac{m_\rho m_{D_s}}{K_\rho^3} \frac{\Gamma(\rho^0 \rightarrow \pi^+\pi^-)}{(1 - 4m_\pi^2/m_\rho^2)^{3/2}} J \quad (26)$$

The value of J obtained by numerical integration is $J = 8.0997$ and we get

$$\frac{\Gamma(D_s^+ \rightarrow \rho^0\pi^+ \rightarrow \pi^+\pi^-\pi^+)}{\Gamma(D_s^+ \rightarrow \rho^0\pi^+)} = 0.9719 \pm 0.0096 \quad (27)$$

where the error is due to the uncertainty of the ρ^0 meson width.

Finite ρ^0 meson width effect is small and the upper limits obtained in Eq. (20) for $|f_{\pi'}|$ are only slightly modified

$$|f_{\pi'}| < \begin{vmatrix} 11.61 \\ 13.30 \\ 17.98 \end{vmatrix} \text{ MeV } \text{ for } \Gamma_{\pi'} = \begin{vmatrix} 600 \\ 400 \\ 200 \end{vmatrix} \text{ MeV} \quad (28)$$

IV. The decay mode for $D_s^+ \rightarrow f_0\pi^+ \rightarrow \pi^+\pi^+\pi^-$

1). We consider now the quasi two body final state $f_0(980)\pi^+$. Since two body decay amplitude has a mass dimension, let us write the transition matrix element for spinless particles in the form,

$$\langle f_0 \pi^+ | T | D_s^+ \rangle \equiv m_{D_s} g_{D_s f_0 \pi} \quad (29)$$

where the dimensionless quantity $g_{D_s f_0 \pi}$ is a weak complex constant in terms of which, the decay width is given by

$$\Gamma(D_s^+ \rightarrow f_0 \pi^+) = \frac{1}{8\pi} K_f |g_{D_s f_0 \pi}|^2. \quad (30)$$

Here K_f is the C. M. momentum of the final particles $f_0 \pi^+$ in the D_s^+ rest frame.

2). The considerations made in the introduction concerning the quark content of the $f_0(980)$ meson can be tested in the decay of interest here, $D_s^+ \rightarrow f_0 \pi^+$. Let us first assume that $f_0(980)$ is a $(u\bar{u} + d\bar{d})/\sqrt{2}$ bound state, then we can show that this assumption is in contradiction with experimental data. Indeed under such assumption, the only possible mechanism for both decays $D_s \rightarrow f_0 \pi^+$ and $D_s \rightarrow \rho^0 \pi^+$ is the WA and the coupling constant $g_{D_s f_0 \pi}$ is obtained from a diagram similar to the one of Fig.1 where the ρ^0 is replaced by the f_0 . It is straightforward to derive a relation between the two weak coupling constants

$$\frac{(g_{D_s f_0 \pi})_{WA}}{g_{D_s \rho \pi}} = \frac{m_{\pi'}}{m_{D_s}} \frac{g_{\pi' f_0 \pi}}{g_{\pi' \rho \pi}} \quad (31)$$

where the subscript WA indicates that the decay amplitude is performed in the W annihilation model of Fig.1, from which we get

$$\frac{\Gamma(D_s^+ \rightarrow f_0 \pi^+)_{WA}}{\Gamma(D_s^+ \rightarrow \rho^0 \pi^+)} = \left(\frac{K'_\rho}{K_\rho} \right)^3 \left(\frac{K_f}{K'_f} \right) \left(\frac{m'_{\pi}}{m_{D_s}} \right)^2 \frac{\Gamma(\pi' \rightarrow f_0 \pi^+)}{\Gamma(\pi' \rightarrow \rho^0 \pi^+)} \quad (32)$$

where K'_f is the C. M. momentum in the π' rest frame associated to the decay $\pi' \rightarrow f_0 \pi^+$.

We assume that the experimental result of the π' decay given in [4] for the final state $(\pi\pi)_{S\text{-wave}} + \pi^+$ corresponds to $f_0 \pi^+$ and we use

$$\frac{\Gamma(\pi' \rightarrow f_0 \pi^+)}{\Gamma(\pi' \rightarrow \rho^0 \pi^+)} = 2.12. \quad (33)$$

Numerically, we obtain from Eq.(32)

$$\Gamma(D_s^+ \rightarrow f_0 \pi^+)_{WA} = 0.318 \cdot \Gamma(D_s^+ \rightarrow \rho^0 \pi^+). \quad (34)$$

Using the upper limit (10) for the rate of $D_s^+ \rightarrow \rho^0 \pi^+$, we get the prediction

$$\mathcal{B}(D_s^+ \rightarrow f_0 \pi^+)_{WA} < 8.9 \cdot 10^{-4} \quad (35)$$

in complete disagreement with the observed experimental value Eq.(9).

It follows that the dominant component of $f_0(980)$ cannot be $(u\bar{u} + d\bar{d})/\sqrt{2}$ and a large $s\bar{s}$ component must be present. In fact such an $s\bar{s}$ component will be responsible for the observed strong decay mode $f_0 \rightarrow K\bar{K}$.

3). Having shown that $f_0(980)$ is mainly a pure $s\bar{s}$ state by both considerations based firstly on its decay modes into $K\bar{K}$, secondly on those of D_s^+ into $f_0 \pi$ (beside the mixing in the $J^{PC} = 0^{++}$ $SU(3)$

flavor nonet argument mentioned previously in the Introduction), the dominant decay mechanism for the mode $D_s^+ \rightarrow f_0 \pi^+$ is then the spectator diagram. The weak coupling constant $g_{D_s f_0 \pi}$ is now computed from the diagram of Fig.2 and the result is

$$g_{D_s f_0 \pi} = i a_1 \frac{G_F m_{D_s}^2}{\sqrt{2}} V_{cs}^* V_{ud} \frac{(m_{D_s}^2 - m_{f_0}^2)}{m_{D_s}^2} \frac{f_\pi}{m_{D_s}} F_0^{D_s f_0}(m_\pi^2) \quad (36)$$

where $F_0^{D_s f_0}(q^2)$ is the $D_s \rightarrow f_0$ hadronic form factor in the BSW notation associated to the spin zero part of the weak axial vector current. Using the experimental branching ratio $\mathcal{B}(D_s^+ \rightarrow f_0 \pi^+)$ in Eq. (9), we deduce the value of the hadronic form factor $F_0^{D_s f_0}(m_\pi^2)$

$$\left| F_0^{D_s f_0}(m_\pi^2) \right| = 0.36 \pm_{0.08}^{0.06} \quad (37)$$

which appears to be an acceptable order of magnitude for an hadronic form factor.

If the $f_0(980)$ is not a pure $s\bar{s}$ state, what has been obtained numerically in Eq.(37) is the product of $F_0^{D_s f_0}(m_\pi^2)$ by a mixing coefficient μ corresponding to the amount of $s\bar{s}$ contained in $f_0(980)$. Of course, $|\mu| \leq 1$, such that the true form factor is $|F_0^{D_s f_0}(m_\pi^2)| \geq 0.36 \pm_{0.08}^{0.06}$.

It is interesting to compare this phenomenological value with the similar one coming from the well measured $D_s^+ \rightarrow \phi \pi^+$ decay mode. In the ideal mixing situation close to reality, the ϕ meson is a pure $s\bar{s}$ state and the decay $D_s^+ \rightarrow \phi \pi^+$ is described by the spectator mechanism involving the hadronic form factor $A_0^{D_s \phi}(m_\pi^2)$. Using the experimental result quoted in Eq.(6) we obtain an experimental value for $A_0^{D_s \phi}(m_\pi^2)$

$$|A_0^{D_s \phi}(m_\pi^2)| = 0.72 \pm 0.04 \quad (38)$$

Of course this form factor is larger than the one previously obtained for $\mu F_0^{D_s f_0}(m_\pi^2)$, but there are no reasons why they should be equal.

However let us remind that both $F_0^{D_s f_0}$ and $A_0^{D_s \phi}$ are determined by the factorization method. It is possible that non factorization contributions will change these numerical estimates in a sizable way, unfortunately, to our knowledge, a convincing approach to compute the non factorization term is not yet available.

4). We now take into account the f_0 meson width effect. The decay amplitude for $D_s^+ \rightarrow f_0 \pi^+ \rightarrow \pi^+ \pi^- \pi^+$ is obtained with the help of the definition Eq.(29), together with the similar expression associated to the decay $f_0 \rightarrow \pi^+ \pi^-$.

$$\begin{aligned} \langle \pi^+ \pi^- \pi^+ | T | D_s^+ \rangle_{f_0 \pi} &= g_{D_s f_0 \pi} g_{f_0 \pi^+ \pi^-} \frac{m_{D_s}}{m_{f_0}} \\ &\left\{ \frac{m_{f_0}^2}{m_{f_0}^2 - s_1 - i m_{f_0} \Gamma_{f_0}(s_1)} + \frac{m_{f_0}^2}{m_{f_0}^2 - s_2 - i m_{f_0} \Gamma_{f_0}(s_2)} \right\} \quad (39) \end{aligned}$$

and the corresponding decay rate is

$$\Gamma(D_s^+ \rightarrow f_0 \pi^+ \rightarrow \pi^+ \pi^- \pi^+) = \frac{1}{64 \pi^3} \frac{m_{D_s}^3}{m_{f_0}^2} |g_{D_s f_0 \pi}|^2 g_{f_0 \pi^+ \pi^-}^2 \frac{K}{2} \quad (40)$$

where the dimensionless coefficient K is the phase space integral

$$K = \frac{m_{f_0}^4}{m_{D_s}^2} \int \int dE_1 dE_2 \left| \frac{1}{m_{f_0}^2 - s_1 - i \sqrt{s_1} \Gamma_{f_0}} + \frac{1}{m_{f_0}^2 - s_2 - i \sqrt{s_2} \Gamma_{f_0}} \right|^2 \quad (41)$$

The energy dependence of the f_0 meson width has been taken as explained previously and the factor $\frac{1}{2}$ in Eq.(40) is due to the presence of two π^+ in the final state.

The dimensionless constant $g_{f_0 \pi^+ \pi^-}$ is related to the partial decay width by

$$\Gamma(f_0 \rightarrow \pi^+ \pi^-) = \frac{1}{8 \pi} K_\pi (g_{f_0 \pi^+ \pi^-})^2 \quad (42)$$

where K_π is the C.M. momentum of the two pions in the f_0 rest frame.

Combining Eqs.(40) and (42) we obtain

$$\Gamma(D_s^+ \rightarrow f_0 \pi^+ \rightarrow \pi^+ \pi^- \pi^+) = \frac{1}{8 \pi^2} \left(\frac{m_{D_s}}{m_{f_0}} \right)^3 \frac{\Gamma(f_0 \rightarrow \pi^+ \pi^-)}{\sqrt{1 - 4m_\pi^2/m_{f_0}^2}} K |g_{D_s f_0 \pi}|^2 \quad (43)$$

The effect of the non zero f_0 width is estimated by comparing Eqs.(30) and (43)

$$\frac{\Gamma(D_s^+ \rightarrow f_0 \pi^+ \rightarrow \pi^+ \pi^- \pi^+)}{\Gamma(D_s^+ \rightarrow f_0 \pi^+) \times \mathcal{B}(f_0 \rightarrow \pi^+ \pi^-)} = \frac{1}{\pi} \left(\frac{m_{D_s}}{m_{f_0}} \right)^3 \frac{\Gamma_{f_0}}{\sqrt{1 - 4m_\pi^2/m_{f_0}^2}} \frac{K}{K_f} \quad (44)$$

The value of K obtained by numerical integration is $K = 5.7217$. We get

$$\frac{\Gamma(D_s^+ \rightarrow f_0 \pi^+ \rightarrow \pi^+ \pi^- \pi^+)}{\Gamma(D_s^+ \rightarrow f_0 \pi^+) \times \mathcal{B}(f_0 \rightarrow \pi^+ \pi^-)} = 0.9886 \pm 0.1893 \quad (45)$$

where the error is due to the uncertainty on the f_0 meson width taken to be $\Gamma_{f_0} = 47 \pm 9 \text{ MeV}$.

The f_0 finite width effect is small and the value of $F_0^{D_s f_0}(m_\pi^2)$ obtained in Eq.(37) is only slightly modified.

V. Total rate for the decay $D_s^+ \rightarrow \pi^+ \pi^+ \pi^-$

1). We propose a model where the decay amplitude $\langle \pi^+ \pi^- \pi^+ | T | D_s^+ \rangle$ is written as a sum of three contributions

$$\begin{aligned} \langle \pi^+ \pi^- \pi^+ | T | D_s^+ \rangle &= \langle \pi^+ \pi^- \pi^+ | T | D_s^+ \rangle_{\rho\pi} + \langle \pi^+ \pi^- \pi^+ | T | D_s^+ \rangle_{f_0\pi} \\ &+ \langle \pi^+ \pi^- \pi^+ | T | D_s^+ \rangle_{NR} \end{aligned} \quad (46)$$

The two first components in Eq.(46) are associated to the quasi two body final states $\rho^0\pi^+$ and $f_0\pi^+$ and their expressions have been given in Eqs.(21) and (39). The third component describes a non resonant $\pi^+\pi^-\pi^+$ state and is naturally assumed to proceed through the W^+ annihilation mechanism. In this case the general structure of the decay matrix element is

$$\langle \pi^+\pi^-\pi^+ | T | D_s^+ \rangle_{WA} = i a_1 \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} f_{D_s} P_{D_s}^\mu \langle \pi^+\pi^-\pi^+ | \mathcal{A}_\mu | 0 \rangle \quad (47)$$

The matrix element of the divergence of the weak axial vector current between the vacuum and the three pion final state is an unknown structure function $F(E_1, E_2)$ depending on two independent variables, choosen as the pion energies.

$$P_{D_s}^\mu \langle \pi^+\pi^-\pi^+ | \mathcal{A}_\mu | 0 \rangle = m_{D_s} F(E_1, E_2) \quad (48)$$

Since the matrix element of the three body decay is dimensionless, Eq.(47) implies that the function $F(E_1, E_2)$ is also dimensionless and is related to the function F_4 of [5] by $F(E_1, E_2) = m_{D_s} F_4(s_1, s_2, Q^2 = m_{D_s}^2)$.

Combining Eqs.(47) and (48), we get

$$\langle \pi^+\pi^-\pi^+ | T | D_s^+ \rangle_{WA} = i a_1 \frac{G_F m_{D_s}^2}{\sqrt{2}} V_{cs}^* V_{ud} \frac{f_{D_s}}{m_{D_s}} F(E_1, E_2) \quad (49)$$

The corresponding branching ratio is written in the form

$$\mathcal{B}(D_s^+ \rightarrow \pi^+\pi^-\pi^+)_{WA} = \mathcal{N} \times I \quad (50)$$

where \mathcal{N} is a normalization factor,

$$\mathcal{N} = \frac{\tau_{D_s}}{\hbar} \frac{m_{D_s}}{64 \pi^3} \frac{1}{2} a_1^2 \left(\frac{G_F m_{D_s}^2}{\sqrt{2}} \right) |V_{cs}|^2 |V_{ud}|^2 \frac{f_{D_s}^2}{m_{D_s}^2} \quad (51)$$

and I a phase space integral given by

$$I = \frac{1}{m_{D_s}^2} \int \int dE_1 dE_2 |F(E_1, E_2)|^2 \quad (52)$$

In Eq.(51) the factor $\frac{1}{2}$ is again due to the presence of the two π^+ in the final state.

The value of \mathcal{N} is computed using the same parameters as in section III and the result is $\mathcal{N} = 1.0399 \cdot 10^{-2}$.

2). In the non resonant case we assume the structure function F to be a constant independent of the pion energies such as $F(E_1, E_2) = F_{NR}$. The experimental non resonant rate quoted in Eq.(8) allows us to extract the modulus of F_{NR} . The corresponding phase space integral I_{NR} in Eq.(52) with constant F_{NR} has the value

$$I_{NR} = 0.1053 |F_{NR}|^2 \quad (53)$$

and we obtain by using Eqs.(8), (50) and (53)

$$|F_{NR}| = 3.04 \pm_{0.58}^{0.49} \quad (54)$$

3). A decomposition formally similar to Eq.(49) in terms of $F(E_1, E_2)$ can be conveniently written for the two other components of Eq.(46). This is obvious for the $\rho\pi$ component which is of the W annihilation type and we get from Eqs.(13) and (21)

$$F_{\rho\pi}(E_1, E_2) = A_{\rho\pi} \cdot H_{\rho\pi}(E_1, E_2) \quad (55)$$

with

$$A_{\rho\pi} = -i \frac{m_{D_s} f_{\pi'}}{m_{\pi'}^2 - m_{D_s}^2 - i m_{\pi'} \Gamma_{\pi'}(m_{D_s}^2)} g_{\pi'\rho\pi} g_{\rho\pi\pi} = |A_{\rho\pi}| e^{i\phi_{\rho\pi}} \quad (56)$$

$$H_{\rho\pi}(E_1, E_2) = \frac{s_2 - s_3}{m_\rho^2 - s_1 - i\sqrt{s_1}\Gamma_\rho} + \frac{s_1 - s_3}{m_\rho^2 - s_2 - i\sqrt{s_2}\Gamma_\rho} \quad (57)$$

For the $f_0\pi$ component which is of the spectator type we obtain, from Eqs.(36) and (39), the structure function $F_{f_0\pi}(E_1, E_2)$ rewritten in terms of the form factor $F_0^{D_s f_0}(m_\pi^2)$

$$F_{f_0\pi}(E_1, E_2) = A_{f_0\pi} H_{f_0\pi}(E_1, E_2) \quad (58)$$

with

$$A_{f_0\pi} = \left(\frac{m_{D_s}}{m_{f_0}}\right) \left(\frac{f_\pi}{f_{D_s}}\right) \left(1 - \frac{m_{f_0}^2}{m_{D_s}^2}\right) F_0^{D_s f_0}(m_\pi^2) g_{f_0\pi^+\pi^-} = |A_{f_0\pi}| e^{i\phi_{f_0\pi}} \quad (59)$$

$$H_{f_0\pi}(E_1, E_2) = \frac{m_{f_0}^2}{m_{f_0}^2 - s_1 - i\sqrt{s_1}\Gamma_{f_0}} + \frac{m_{f_0}^2}{m_{f_0}^2 - s_2 - i\sqrt{s_2}\Gamma_{f_0}} \quad (60)$$

In the spectator model the form factor is real, such that $\phi_{f_0\pi} = 0$ or π .

The total structure function $F(E_1, E_2)$ has three components

$$F(E_1, E_2) = F_{\rho\pi}(E_1, E_2) + F_{f_0\pi}(E_1, E_2) + F_{NR} \quad (61)$$

and the computation of the total rate is made using the formula

$$\mathcal{B}(D_s^+ \rightarrow \pi^+\pi^-\pi^+) = 1.0399 \cdot 10^{-2} I \quad (62)$$

where I is the phase space integral defined in Eqs.(52).

Let us first consider the phase space integrals involving the functions $H(E_1, E_2)$. We have previously obtained

$$\frac{1}{m_{D_s}^2} \int \int dE_1 dE_2 = 0.1053 \quad (63)$$

$$\frac{1}{m_{D_s}^2} \int \int |H_{f_0\pi}(E_1, E_2)|^2 dE_1 dE_2 = K = 5.7217 \quad (64)$$

$$\frac{1}{m_{D_s}^2} \int \int |H_{\rho\pi}(E_1, E_2)|^2 dE_1 dE_2 = J = 8.0997 \quad (65)$$

For the interferences between the three components we need three more integrals

$$H_{f_0\pi} = \frac{1}{m_{D_s}^2} \int \int H_{f_0\pi}(E_1, E_2) dE_1 dE_2 = |H_{f_0\pi}| e^{i\phi_{H_{f_0\pi}}} \quad (66)$$

$$H_{\rho^0\pi} = \frac{1}{m_{D_s}^2} \int \int H_{\rho\pi}(E_1, E_2) dE_1 dE_2 = |H_{\rho\pi}| e^{i\phi_{H_{\rho\pi}}} \quad (67)$$

$$H_{\rho f} = \frac{1}{m_{D_s}^2} \int \int H_{\rho\pi}^*(E_1, E_2) H_{f_0\pi}(E_1, E_2) dE_1 dE_2 = |H_{\rho f}| e^{i\phi_{H_{\rho f}}} \quad (68)$$

The numerical results are

$$|H_{f_0\pi}| = 0.2677 \quad \phi_{H_{f_0\pi}} = 89.6^0 \quad (69)$$

$$|H_{\rho\pi}| = 5.58 \cdot 10^{-5} \quad \phi_{H_{\rho\pi}} = -84.5^0 \quad (70)$$

$$|H_{\rho f}| = 0.3868 \quad \phi_{H_{\rho f}} = 241.5^0 \quad (71)$$

The numerical values of $|F_{NR}|$ and $|A_{f_0\pi}|$ are taken from the measured rates respectively in Eqs.(8) and (9). In the $\rho\pi$ case we write $A_{\rho\pi} = \lambda A_{\rho\pi}^{max}$ with $0 \leq \lambda < 1$ where $\lambda = 1$ corresponds to the 90% C. L. experimental limit in Eq.(10) .

There is à priori no reason for the non resonant amplitude F_{NR} to be real and we allow an arbitrary phase ϕ_{NR} . The two other phases $\phi_{f_0\pi}$ and $\phi_{\rho\pi}$ are determined by our model as

$$\phi_{\rho\pi} = (70 \pm 10)^0 \quad for \quad \Gamma_{\pi'} = (400 \pm 200) MeV \quad (72)$$

$$\phi_{f_0\pi} = 0^0 \quad for \quad F_0^{D_s f_0}(m_\pi^2) > 0. \quad (73)$$

The formula for $I(\pi^+\pi^-\pi^+)_{TOT}$ is then written as

$$\begin{aligned} I(\pi^+\pi^-\pi^+)_{TOT} &= I(\pi^+\pi^-\pi^+)_{NR} + I(\pi^+\pi^-\pi^+)_{f_0\pi} + \lambda^2 I(\pi^+\pi^-\pi^+)_{\rho\pi}^{max} \\ &+ 2 |F_{NR}| |A_{f_0\pi}| |H_{f_0\pi}| \cos[\phi_{f_0\pi} + \phi_{H_{f_0\pi}} - \phi_{NR}] \\ &+ 2 \lambda |A_{\rho\pi}|^{max} |A_{f_0\pi}| |H_{\rho f}| \cos[\phi_{f_0\pi} - \phi_{\rho\pi} + \phi_{H_{\rho\pi}}] \\ &+ 2 \lambda |F_{NR}| |A_{\rho\pi}|^{max} |H_{\rho\pi}| \cos[\phi_{\rho\pi} + \phi_{H_{\rho\pi}} - \phi_{NR}] \end{aligned} \quad (74)$$

The errors for $I(\pi^+\pi^-\pi^+)_{TOT}$ are computed in quadrature from the experimental errors on $I(\pi^+\pi^-\pi^+)_{NR}$ and $I(\pi^+\pi^-\pi^+)_{f_0\pi}$ and for the errors of the amplitudes $|F_{NR}|$ and $|A_{f_0\pi}|$, we use the approximate relation $\Delta x \simeq \frac{1}{2x} \Delta x^2$.

The result of our calculation of $I(\pi^+\pi^-\pi^+)_{TOT}$ directly gives the total branching ratio by the relation (62) and is represented in Figs.3 as a function of the phase parameter ϕ_{NR} for the two

extreme cases $\lambda = 0$ and $\lambda = 1$. The one standard deviation domains can be compared with the experimental result of Eq.(7).

It is clear from Fig.3-a corresponding to a two component model ($\lambda = 0$), and Fig.3-b where a maximal $\rho^0\pi^+$ contribution has been added ($\lambda = 1$), that there is no difficulty in fitting the experimental result (7) within one standard deviation, for any value of the phase parameter ϕ_{NR} .

VI. Energy distributions of the pion in $D_s^+ \rightarrow \pi^+\pi^+\pi^-$ decay

1). The two π^+ meson energies being E_1 and E_2 with our convention of section II, the function $F(E_1, E_2)$ introduced in section V is symmetrical in the exchange of E_1 and E_2 . Such a property extends to the Dalitz plot which is a double differential distribution proportional to $|F(E_1, E_2)|^2$. However it is probably premature to discuss the detailed properties of the Dalitz plot and it is more realistic to consider single differential quantities like the π meson energy distributions.

We must keep in mind that a distribution in the pion energy E_j is just the mirror of a distribution in the invariant two pion mass squared s_j because of the relations (1), $s_j = m_{D_s}^2 + m_\pi^2 - 2m_{D_s}E_j$.

2). For the final state $\pi^+\pi^+\pi^-$, we define the π^+ and π^- energy distributions in the following way

$$\frac{d\Gamma(D_s^+ \rightarrow \pi^+\pi^+\pi^-)}{dE_1} = \tilde{\mathcal{N}} G_+(E_1) \quad , \quad \frac{d\Gamma(D_s^+ \rightarrow \pi^+\pi^+\pi^-)}{dE_3} = \tilde{\mathcal{N}} G_-(E_3) \quad (75)$$

where the distributions $G_+(E_1)$ and $G_-(E_3)$ are defined by

$$G_+(E_1) = \frac{1}{m_{D_s}} \int_{E_-(E_1)}^{E_+(E_1)} |F(E_1, E_2)|^2 dE_2 \quad (76)$$

$$G_-(E_3) = \frac{1}{m_{D_s}} \int_{E_-(E_3)}^{E_+(E_3)} |F(m_{D_s} - E_2 - E_3, E_2)|^2 dE_2 \quad (77)$$

and the limits of integration $E_\pm(E)$ have been given in Eq.(5) of section II.

The normalization constants $\tilde{\mathcal{N}}$ and \mathcal{N} are simply related by

$$\mathcal{N} = \frac{\tau_{D_s}}{\hbar} m_{D_s} \tilde{\mathcal{N}} \quad (78)$$

and the numerical value of $\tilde{\mathcal{N}}$ is $\tilde{\mathcal{N}} = 7.446 \cdot 10^{-15}$.

The π^+ energy distribution $G_+(E_1)$ and the π^- energy distribution $G_-(E_3)$ for all $\pi^+\pi^+\pi^-$ events have been computed performing a single variable integration with the relevant functions defined in section V, $|H_{f_0\pi}(E_1, E_2)|^2$, $|H_{\rho\pi}(E_1, E_2)|^2$, $H_{f_0\pi}(E_1, E_2)$, $H_{\rho\pi}(E_1, E_2)$, and $H_{\rho\pi}^*(E_1, E_2)H_{f_0\pi}(E_1, E_2)$. We observe that the last three integrals are complex with an energy dependent phase and, as a consequence, the nature of constructive or destructive of the various interferences is an energy dependent

concept. The terms F_{NR} , $A_{f_0\pi}$, and $A_{\rho\pi}$ are treated as in section V with their moduli extracted from the measured branching ratios. The two extreme cases $\lambda = 0$ and $\lambda = 1$ have been considered.

It is clear that the shape of the π meson energy distributions $G_+(E_1)$ and $G_-(E_3)$ depends on the constant phase of the non resonant amplitude F_{NR} in a very sensitive way. We notice in [1] that we have a Dalitz plot in the (s_1, s_2) plane corresponding to the (68.1 ± 12.4) events observed in $D_s^+ \rightarrow \pi^+\pi^-\pi^+$ decays. A projection of this Dalitz plot on the s_1 axis shows clearly a peak associated to the $f_0\pi^+$ quasi two body state and an important background. Of course, such an histogram in [1] is nothing but our π^+ meson energy distribution $G_+(E_1)$.

The $G_+(E_1)$ distribution has been studied for values of ϕ_{NR} between 0^0 and 360^0 . Comparing our theoretical curves with the shape of the experimental histogram [1], we obtain an acceptable agreement if the angle ϕ_{NR} is in the range

$$70^0 < \phi_{NR} < 140^0 \quad (79)$$

Let us now come back to Eq.(49) where the function $F(E_1, E_2)$ is defined. Beside the phase of the CKM factors, we have the phase $\pi/2$ due to the factor i entering in the definition of the matrix element $< 0 | \mathcal{A}_\mu | D_s^+ >$ and the phase of the function $F(E_1, E_2)$. In the non resonant component, the last phase is ϕ_{NR} . We observe that the value $\phi_{NR} = 90^0$ compatible with the range (79) makes the quantity $i F_{NR}$ real and negative. In the absence of more detailed experimental informations we shall make a simple choice $\phi_{NR} = 90^0$ in what follows.

We have represented in Figs.4 and Figs.5 the pion energy distributions $G_+(E_1)$ and $G_-(E_3)$ for the two cases $\lambda = 0$ and $\lambda = 1$ using $\phi_{NR} = 90^0$. For completeness we give the theoretical branching ratios corresponding to $\phi_{NR} = 90^0$ which can be directly extracted from Figs.3.

$$(i) \quad \lambda = 0, \quad \mathcal{B}(D_s^+ \rightarrow \pi^+\pi^-\pi^+)_{TH} = 2.022 \pm 0.528 \%, \quad (80)$$

$$(ii) \quad \lambda = 1, \quad \mathcal{B}(D_s^+ \rightarrow \pi^+\pi^-\pi^+)_{TH} = 2.259 \pm 0.524 \%. \quad (81)$$

Both values Eqs.(80) and (81) agree with the experimental one of Eq.(7), $(1.54 \pm 0.42) \%$.

VII. Predictions for $D_s^+ \rightarrow \pi^0\pi^0\pi^+$ decay mode.

We have no experimental data on the decay mode $D_s^+ \rightarrow \pi^0\pi^0\pi^+$. However we shall use the same three component model for this decay mode and let us start our discussion with isospin considerations concerning these three components.

1). Consider first the case of a non resonant $\pi^0\pi^0\pi^+$ state. The constant F_{NR} has no reason to be the same for the two modes $\pi^+\pi^-\pi^+$ and $\pi^0\pi^0\pi^+$. By assumption, with a constant function

$F_{NR}(E_1, E_2)$, we have a full symmetry in space between the three pions. From the Bose-Einstein symmetry, the isospin configuration has also to be totally symmetric. Consider now a third rank fully symmetric tensor in a three dimensional space. It has 10 independent components. With respect to the isospin $SO(3)$ orthogonal group, such a tensor is reducible into an isospin $I = 3$ part with 7 components and an isospin $I = 1$ part with 3 components. In our specific W annihilation model for the non resonant three pions, only the later part contributes, the $u\bar{d}$ weak current being an isovector. By inspection of the relevant Clebsch-Gordan coefficients, we get the result

$$F_{NR}(\pi^+\pi^+\pi^-) = 3 F_{NR}(\pi^0\pi^0\pi^+) \quad . \quad (82)$$

As a consequence, in our model, we obtain for the non resonant part :

$$\frac{\Gamma(D_s^+ \rightarrow \pi^0\pi^0\pi^+)_{NR}}{\Gamma(D_s^+ \rightarrow \pi^+\pi^+\pi^-)_{NR}} = \frac{1}{9} = 11.1 \quad \% \quad , \quad (83)$$

and the non resonant branching ratio $\mathcal{B}(D_s^+ \rightarrow \pi^0\pi^0\pi^+)_{NR}$ is expected to occur only at the 10^{-3} level.

2). The quasi two body state $\rho^0\pi^+$ cannot produce a $\pi^0\pi^0\pi^+$ final state because of the isovector character of the ρ meson. However $\rho^+\pi^0$ can give such a state when followed by the decay $\rho^+ \rightarrow \pi^+\pi^0$. The D_s^+ meson being an isoscalar, the decay amplitudes for $D_s \rightarrow \rho^0\pi^+$ and $D_s \rightarrow \rho^+\pi^0$ are equal. When mass differences between charged and neutral ρ 's and π 's are neglected we obtain

$$\Gamma(D_s^+ \rightarrow \rho^0\pi^+) = \Gamma(D_s^+ \rightarrow \rho^+\pi^0) \quad (84)$$

With equal decay widths for $\rho^0 \rightarrow \pi^+\pi^-$ and $\rho^+ \rightarrow \pi^+\pi^0$ we obtain the ratio of rates

$$\frac{\Gamma(D_s^+ \rightarrow \rho^+\pi^0 \rightarrow \pi^+\pi^0\pi^0)}{\Gamma(D_s^+ \rightarrow \rho^0\pi^+ \rightarrow \pi^+\pi^-\pi^+)} = 1 \quad (85)$$

We observe that the finite ρ width corrections are the same in both final states $\pi^+\pi^-\pi^+$ and $\pi^0\pi^0\pi^+$ because two Breit Wigner amplitudes are present in the variables s_1 and s_2 with the choice made in section II.

3). For the third component, i.e., the decay $D_s^+ \rightarrow f_0\pi^+$ followed by $f_0 \rightarrow \pi^+\pi^-$ ($f_0 \rightarrow \pi^0\pi^0$) produces a $\pi^+\pi^-\pi^+(\pi^0\pi^0\pi^+)$ final state. Because of the isoscalar character of the $f_0(980)$ we have

$$\Gamma(f_0 \rightarrow \pi^+\pi^-) = 2 \Gamma(f_0 \rightarrow \pi^0\pi^0) \quad (86)$$

On the other hand, in the $D_s^+ \rightarrow f_0\pi^+ \rightarrow \pi^+\pi^-\pi^+$ case, we have two Breit-Wigner amplitudes in the variables s_1 and s_2 whereas in the $D_s^+ \rightarrow f_0\pi^+ \rightarrow \pi^0\pi^0\pi^+$ case there exists only one Breit-Wigner amplitude in the variable s_3 with the choice made in section II. As a consequence the phase

space integral K introduced in Eq.(41) has the value $K_c = 5.7217$ for the final state $\pi^+\pi^-\pi^+$ and $K_N = 2.7469$ for the final state $\pi^0\pi^0\pi^+$, and we obtain the ratio of rates

$$\frac{\Gamma(D_s^+ \rightarrow f_0\pi^+ \rightarrow \pi^0\pi^0\pi^+)}{\Gamma(D_s^+ \rightarrow f_0\pi^+ \rightarrow \pi^+\pi^-\pi^+)} = \frac{1}{2} \frac{K_N}{K_c} = 24\% \quad (87)$$

The departure of this ratio from $\frac{1}{4}$ is simply due to the interference between the two Breit-Wigner contributions in the $D_s^+ \rightarrow f_0\pi^+ \rightarrow \pi^+\pi^-\pi^+$ case.

4). The function $F(E_1, E_2)$ associated to the final state $\pi^0\pi^0\pi^+$ is written in the form

$$F(E_1, E_2) = \frac{1}{3}F_{NR} + \frac{1}{\sqrt{2}}A_{f_0\pi} H_{f_0\pi}(E_1, E_2) + \lambda A_{\rho\pi}^{max} H_{\rho\pi}(E_1, E_2) \quad (88)$$

where in the right hand side of Eq.(88) all terms but $H_{f_0\pi}(E_1, E_2)$ are the same as those defined in section V for the $\pi^+\pi^-\pi^+$ final state. In Eq.(88), E_1 and E_2 are the π^0 energies and E_3 the π^+ energy. Whereas $H_{f_0\pi}(E_1, E_2)$ in the $\pi^+\pi^-\pi^+$ case contains two Breit-Wigner terms with the variables s_1 and s_2 (Eq.(60)), for the $\pi^0\pi^0\pi^+$ final state we have only one Breit-Wigner term with the variable s_3

$$H_{f_0\pi}(E_1, E_2) = \frac{m_{f_0}^2}{m_{f_0}^2 - s_3 - i\sqrt{s_3}\Gamma_{f_0}} \quad (89)$$

The total rate is now computed by integration the quantity $|F(E_1, E_2)|$ over E_1 and E_2 . The procedure of calculation is similar to the one explained in detail in section V and the result for $I(\pi^0\pi^0\pi^+)_{TOT}$ is shown on Figs.6 for the two extreme cases $\lambda = 0$ and $\lambda = 1$. Because of the isospin factor $\frac{1}{3}$ in front of F_{NR} the role played by the non resonant component is obviously less important in the $\pi^0\pi^0\pi^+$ case than it was in the $\pi^+\pi^-\pi^+$ one. For such a reason the quantity $I(\pi^0\pi^0\pi^+)_{TOT}$ in the two component model, $\lambda = 0$, is insensitive to the phase parameter ϕ_{NR} . Again due to the isospin factors, the possible role of the quasi two body state $\rho\pi$ might be more important in the $\pi^0\pi^0\pi^+$ case than it was for the $\pi^+\pi^-\pi^+$ one. Such a qualitative expectation is clearly seen in Figs.6. Both Figs.3 and 6 are drawn at the same scale and the order of magnitude of the branching ratio $\mathcal{B}(D_s^+ \rightarrow \pi^0\pi^0\pi^+)$ is expected to be few 10^{-3} and that might be the reason why the mode $D_s^+ \rightarrow \pi^0\pi^0\pi^+$ has not yet been experimentally observed.

With the choice $\phi_{NR} = 90^\circ$ we find

$$(i) \quad \lambda = 0, \quad \mathcal{B}(D_s^+ \rightarrow \pi^0\pi^0\pi^+)_{TH} = 0.294 \pm 0.092 \%, \quad (90)$$

$$(ii) \quad \lambda = 1, \quad \mathcal{B}(D_s^+ \rightarrow \pi^0\pi^0\pi^+)_{TH} = 0.604 \pm 0.101 \%. \quad (91)$$

5). The π^0 and π^+ energy distributions $G_0(E_1)$ and $G_+(E_3)$ for the final state $\pi^0\pi^0\pi^+$ are computed with the formulae (76) and (77) using now the function $F(E_1, E_2)$ given in Eq.(88). The procedure of calculation is similar to the one explained in section VI for the $\pi^+\pi^-\pi^+$ final state.

We have represented respectively in Figs.7 and 8 the π^0 and π^+ energy distributions $G_0(E_1)$ and $G_+(E_3)$ for the two extreme situations $\lambda = 0$ and $\lambda = 1$, using as previously the phase $\phi_{NR} = 90^\circ$ for the non resonant amplitude. Figs.8 show clearly the peak due to the quasi two body $f_0\pi^+$ state and a possible quasi two body $\rho^+\pi^0$ contribution is clearly seen in Fig.7-b.

Let us end by considering the π meson energy distribution corresponding to an uniformly populated phase space of a constant amplitude. The energy distributions for each pion are obviously identical and in the case $D_s^+ \rightarrow (\pi^+\pi^-\pi^+)_{NR}$ we obtain, using Eq.(5), the analytic form

$$G_{NR}(E) = |F_{NR}|^2 \left\{ \frac{(E^2 - m_\pi^2)(m_{D_s}^2 - 3m_\pi^2 - 2m_{D_s}E)}{m_{D_s}^2 + m_\pi^2 - 2m_{D_s}E} \right\}^{1/2} \quad (92)$$

Using the experimental value (54) of $|F_{NR}|^2$, we obtain the distribution represented in Fig.9.

VIII. Summary and Concluding Remarks

We have studied, in this paper, the decay of the D_s^+ meson into three π mesons by analyzing the experimental data available for the decay mode $D_s^+ \rightarrow \pi^+\pi^-\pi^+$. The amplitude can be considered as the superposition of two mechanisms : the spectator decay of the c quark contained in the D_s^+ meson and the W^+ annihilation into three pions. Both mechanisms are needed to explain the data and the latter appears to be considerably important.

The non observation of the $\rho\pi$ quasi two body state allows us to determine an upper limit of about 10 MeV for the leptonic decay constant $f_{\pi'}$ of the $\pi' \equiv \pi(1300)$ meson. Such an upper bound of $f_{\pi'}$ is compatible with theoretical estimates using non-relativistic chiral quark model [10] and QCD sum rule technique [11]. We remark that the commonly used values of the $f_{\pi'}$ decay constant [7] are much larger than our upper limit.

The large $f_0\pi^+$ branching ratio compared to the $\rho^0\pi^+$ one allows us to exclude the dominant $(u\bar{u} + d\bar{d})/\sqrt{2}$ quark structure for the $f_0(980)$ meson. Assuming the $f_0(980)$ to be a $s\bar{s}$ state, the spectator mechanism will govern the decay $D_s^+ \rightarrow f_0(980)\pi^+$ and from experimental data it is possible to estimate the hadronic form factor $F_0^{D_s f_0}(m_\pi^2)$ to be $0.36 \pm_{0.08}^{0.06}$.

Furthermore, a large branching ratio of the D_s^+ meson into a non resonant three pion state has been experimentally measured. This decay can only proceed through the WA and the corresponding decay amplitude is assumed to be independent of the π meson energies. We then introduce a phenomenological complex constant F_{NR} , its modulus is extracted from the non resonant branching ratio $\mathcal{B}(D_s^+ \rightarrow \pi^+\pi^-\pi^+)_{NR}$.

For the full decay amplitude, we propose a three component model involving the non resonant

background, the $f_0\pi^+$ quasi two body state, and a possible $\rho\pi$ quasi two body state with a parameter λ such that for $\lambda = 1$, the branching ratio $\mathcal{B}(D_s^+ \rightarrow \rho^0\pi^+)$ takes the value of the 90% confidence level upper limit. We first compute the total rate for $D_s^+ \rightarrow \pi^+\pi^-\pi^+$ as a function of the phase ϕ_{NR} of the constant amplitude F_{NR} . Agreement with experiment is obtained, within one standard deviation, in both cases $\lambda = 0$ and $\lambda = 1$, for all values of ϕ_{NR} between 0^0 and 360^0 .

For the decay mode $D_s^+ \rightarrow \pi^+\pi^-\pi^+$, we compute the π^+ and π^- energy distributions which depend on the phase parameter ϕ_{NR} . A qualitative agreement between our π^+ distribution with the experimental histogram [1] in the invariant $(\pi^+\pi^-)$ mass is obtained when ϕ_{NR} is restricted in the range given by Eq.(79) including $\phi_{NR} = 90^0$. Fixing now $\phi_{NR} = 90^0$, we present our predictions for the π^+ and π^- energy distributions including one standard deviation errors deduced for those of the experimental rates.

The same model is used to make predictions for the decay mode $D_s^+ \rightarrow \pi^0\pi^0\pi^+$ which is not yet experimentally observed. The total rate for $D_s^+ \rightarrow \pi^0\pi^0\pi^+$ is found to be essentially one order of magnitude smaller than the $D_s^+ \rightarrow \pi^+\pi^-\pi^+$ rate and this might be the reason why the decay $D_s^+ \rightarrow \pi^0\pi^0\pi^+$ has not yet been detected. Also the π^0 and π^+ energy distributions are predicted by our model.

If the quasi two body states $f_0\pi$ and $\rho\pi$ can be theoretically understood, the non resonant part has been treated, in this paper, only from a purely phenomenological view point, using a constant complex parameter F_{NR} . It would be interesting to explain theoretically the experimental values found for $|F_{NR}|$ and ϕ_{NR} . To our knowledge no model has been proposed for such case.

The detailed understanding of the WA is very important in its own right, and for that purpose, the D_s^+ , like the B_c^+ , are particularly auspicious for WA to manifest in its full strength due to both color and CKM favored factors. In all other cases, i.e., D^+, D^0, B^+, B^0 , and B_s^0 , the WA is either masked by the color and CKM suppressed factors, or contaminated by the spectator decay mechanism at the quark level, such that unambiguous WA effect can be hardly isolated. Therefore experimental and theoretical investigations of the D_s^+ (and the B_c^+ later) decays into pions, in both inclusive and exclusive modes, are of great interest. They are the ideal laboratories for studying the weak annihilation mechanism.

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References

- [1] J. C. Anjos et al., Fermi Experiment 691, *Phys. Rev. Lett.* **62** 125 (1989)
- [2] M. Adamovich et al., WA82 Collaboration, *Phys. Lett. B* **305** 117 (1993)
- [3] N. A. Törnqvist, Preprint HU-SEFT R 1995-05
- [4] Review of Particles Properties, Reference as Phys. Rev. **D 50**, August 1994.
- [5] J. H. Kühn and E. Mirkes, *Phys. Lett. B* **286** 381 (1992); *Z. Phys. C* **56** 661 (1992).
- [6] J. Stern, N. H. Fuchs and M. Knecht, Proceedings of the 3rd Workshop on the τ -Charm factory, Preprint IPN/TH 93-38.
- [7] N. Isgur, C. Morningstar and C. Reader, *Phys. Rev. D* **39** 1357 (1989)
- [8] M. Wirbel, B. Stech and M. Bauer, *Z. Phys.* **C29** 637 (1985);
M. Bauer, B. Stech and M. Wirbel, *Z. Phys.* **C34** 103 (1987).
- [9] M. Neubert, V. Rieckert, B. Stech and Q. P. Xu, Advanced Series on Directions in High Energy Physics Vol. 10, Heavy Flavours, page 286, Editors A.J. Buras and M. Lindner, (World Scientific, Singapore, 1992).
- [10] S. Y. Choi, K. Hagiwara, and M. Tanabashi, Preprint KEK-TH-419, KEK Preprint 94-158, November 1994.
- [11] S. Narison, N. Paver, and D. Treleani, *Nuovo Cimento* **74** 347 (1983)

Figure captions

1. **Figure 1 :** W annihilation mechanism for the $D_s^+ \rightarrow \rho^0 + \pi^+$ decay mode.
2. **Figure 2 :** Spectator mechanism for the $D_s^+ \rightarrow f_0 + \pi^+$ decay mode.
3. **Figure 3 :** Total branching ratio for the decay $D_s^+ \rightarrow \pi^+\pi^-\pi^+$ as a function of the phase parameter ϕ_{NR} , (a) in the two component model : $\lambda = 0$, (b) in the three component model with the maximal $\rho\pi^+$ contribution : $\lambda = 1$. One standard deviation errors are indicated and the horizontal bound is the experimental result including one standard deviation errors.
4. **Figure 4 :** The π^+ meson energy distribution function $G_+(E_1)$ for the final state $\pi^+\pi^-\pi^+$, (a) for $\lambda = 0$ in the two component model, (b) for $\lambda = 1$ in the three component model with the maximal $\rho\pi^+$ contribution. One standard deviation errors are indicated. Here the quantity $E_{f_0} = 0.7453 \text{ GeV}$ is associated to the f_0 resonance and $E_\rho = 0.8386 \text{ GeV}$ is associated to the ρ^0 resonance.
5. **Figure 5 :** The π^- meson energy distribution function $G_-(E_3)$ for the final state $\pi^+\pi^-\pi^+$, (a) for $\lambda = 0$ in the two component model, (b) for $\lambda = 1$ in the three component model with the maximal $\rho\pi^+$ contribution. One standard deviation errors are indicated.
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7. **Figure 7 :** The π^0 meson energy distribution function $G_0(E_1)$ for the final state $\pi^0\pi^0\pi^+$, (a) for $\lambda = 0$ in the two component model, (b) for $\lambda = 1$ in the three component model with the maximal $\rho^+\pi^0$ contribution. One standard deviation errors are indicated. Here the quantity $E_\rho = 0.8386 \text{ GeV}$ is associated to the ρ resonance.
8. **Figure 8 :** The π^+ meson energy distribution function $G_+(E_3)$ for the final state $\pi^0\pi^0\pi^+$, (a) for $\lambda = 0$ in the two component model, (b) for $\lambda = 1$ in the three component model with the maximal $\rho^+\pi^0$ contribution. One standard deviation errors are indicated. Here the quantity $E_{f_0} = 0.7453 \text{ GeV}$ is associated to the f_0 resonance.
9. **Figure 9 :** The π meson energy distribution $G_{NR}(E)$ for the non resonant part of the decay $D_s^+ \rightarrow \pi^+\pi^-\pi^+$. One standard deviation errors are indicated.

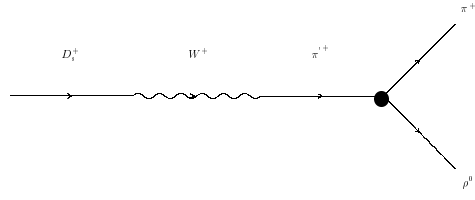


Figure 1: W annihilation mechanism for $D_s^+ \rightarrow \rho^0 + \pi^+$ decay mode.

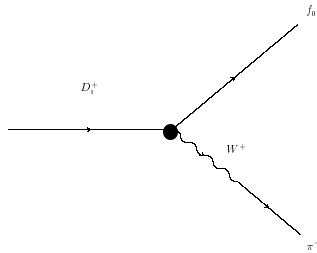


Figure 2: Spectator mechanism for $D_s^+ \rightarrow f_0 + \pi^+$ decay mode.

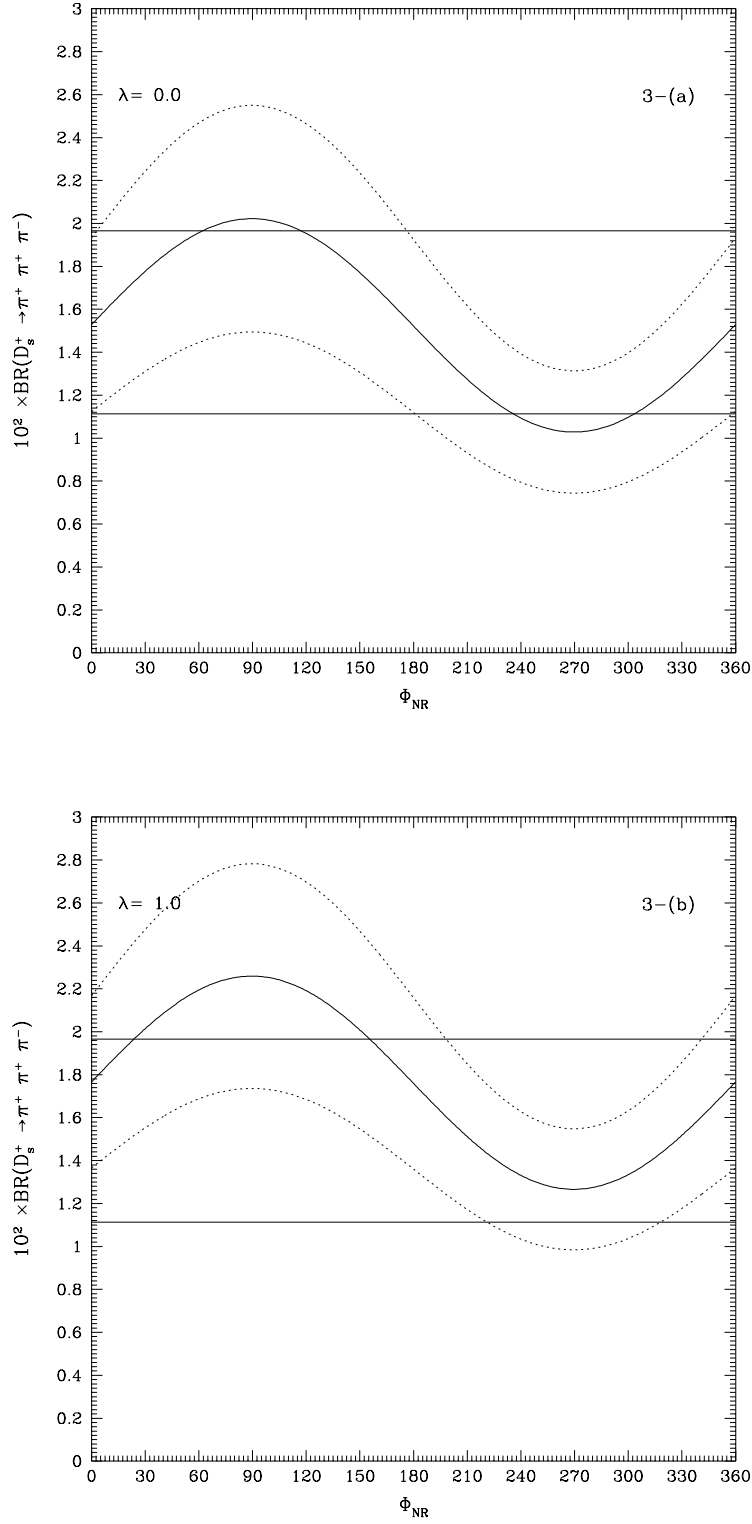


Figure 3: Total branching ratio for the decay $D_s^+ \rightarrow \pi^+ \pi^- \pi^+$ as a function of the phase parameter ϕ_{NR} , (a) in the two component model : $\lambda = 0$, (b) in the three component model with the maximal $\rho\pi^+$ contribution : $\lambda = 1$. One standard deviation errors are indicated and the horizontal bound is the experimental result including one standard deviation errors.

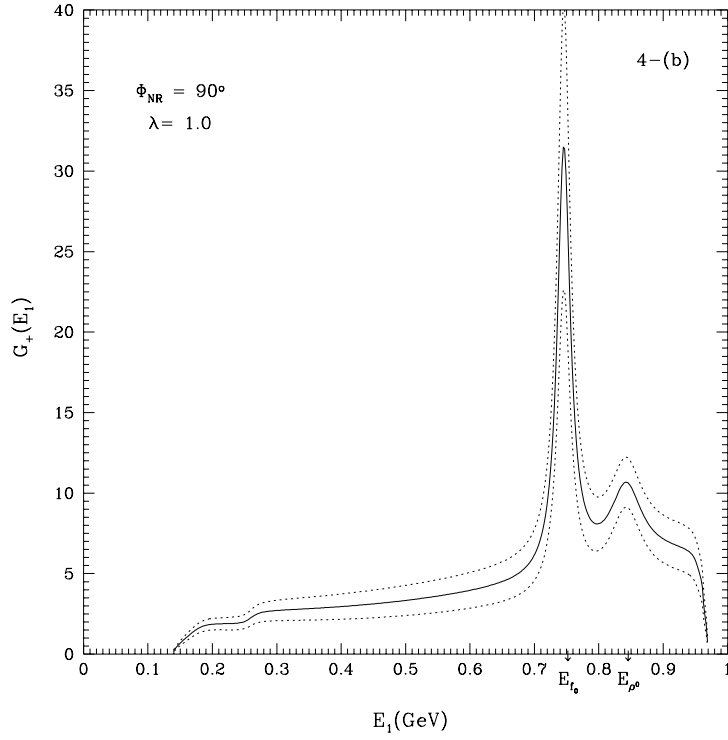
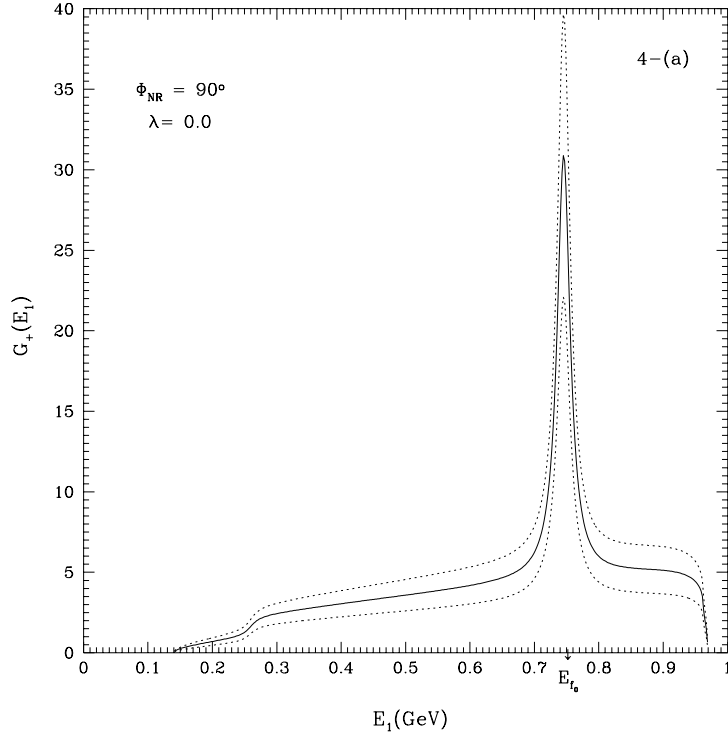


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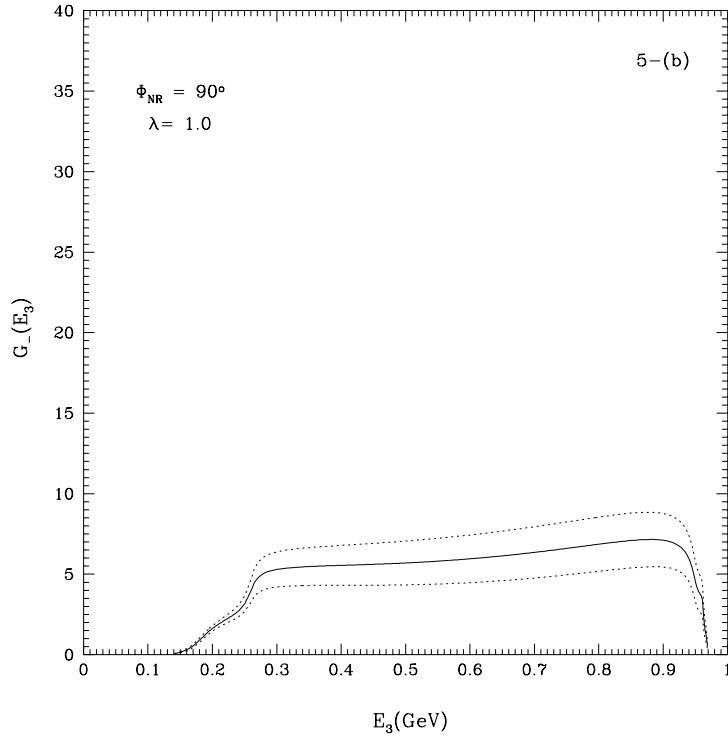
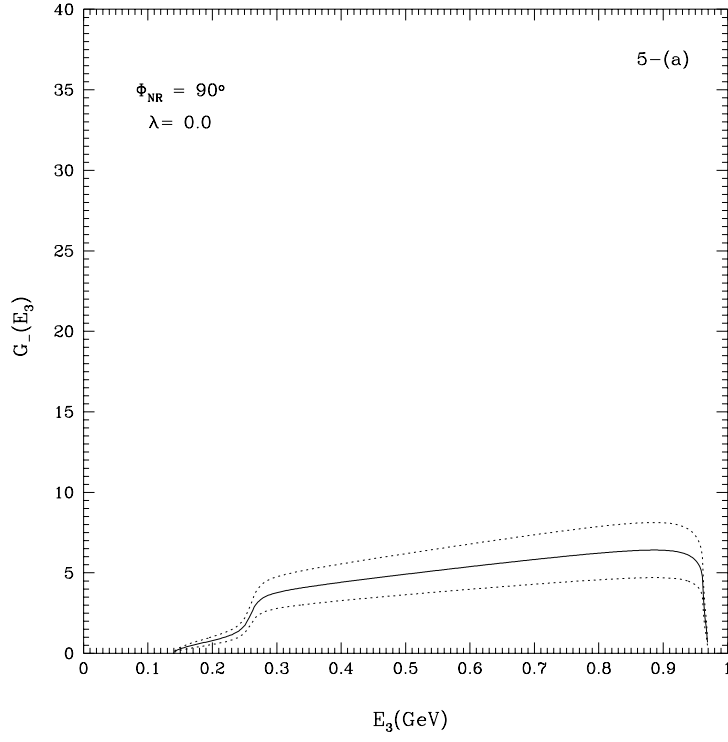


Figure 5: The π^- meson energy distribution function $G_-(E_3)$ for the final state $\pi^+\pi^-\pi^+$, (a) for $\lambda = 0$ in the two component model, (b) for $\lambda = 1$ in the three component model with the maximal $\rho\pi^+$ contribution. One standard deviation errors are indicated.

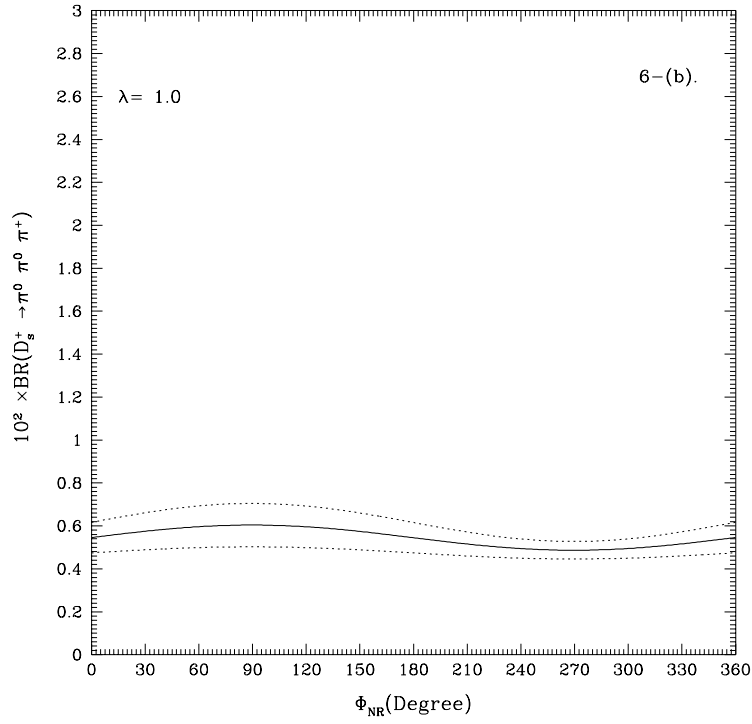
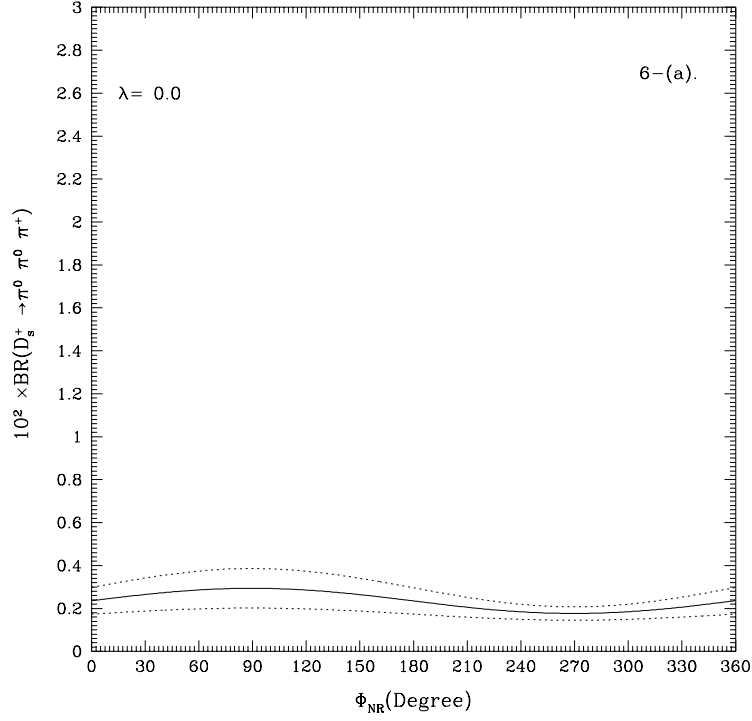


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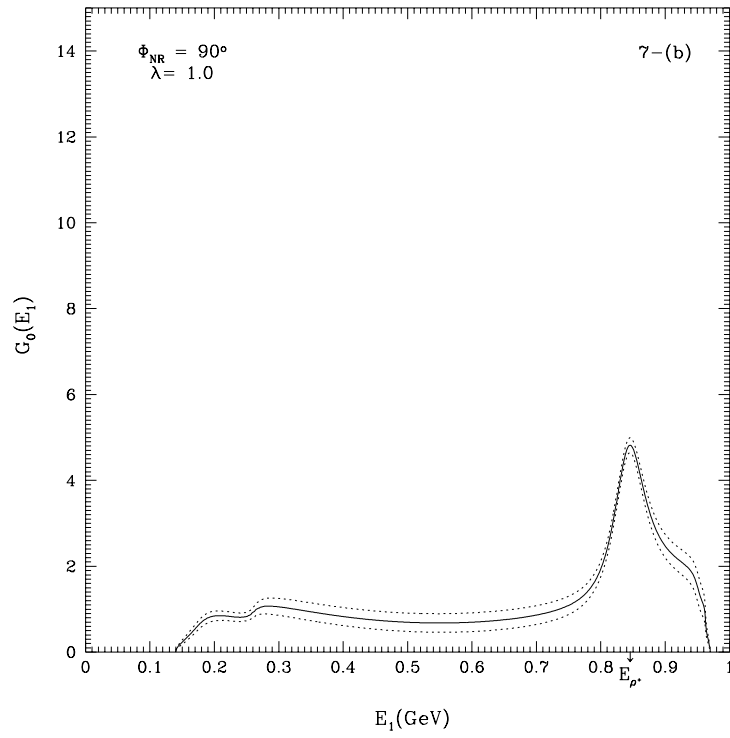
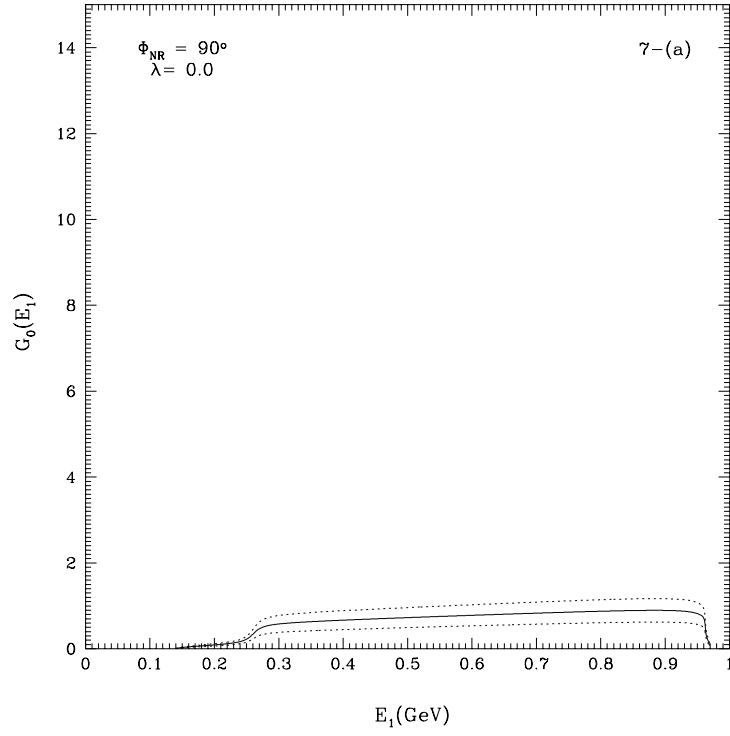


Figure 2: The π^0 meson energy distribution function $G_0(E_1)$ for the final state $\pi^0\pi^0\pi^+$, (a) for $\lambda = 0$ in the two component model, (b) for $\lambda = 1$ in the three component model with the maximal $\rho^+\pi^0$ contribution. One standard deviation errors are indicated. Here the quantity $E_\rho = 0.8386$ GeV is associated to the ρ resonance.

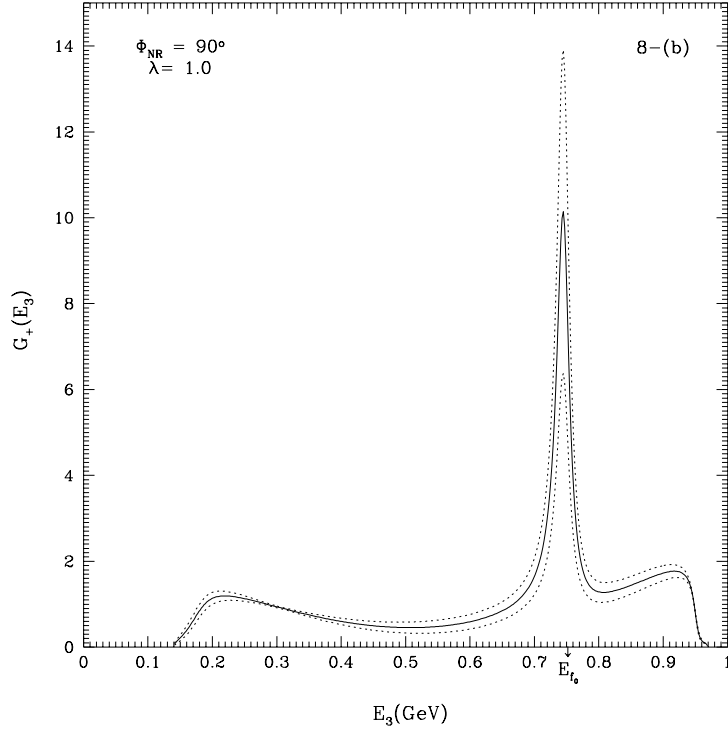
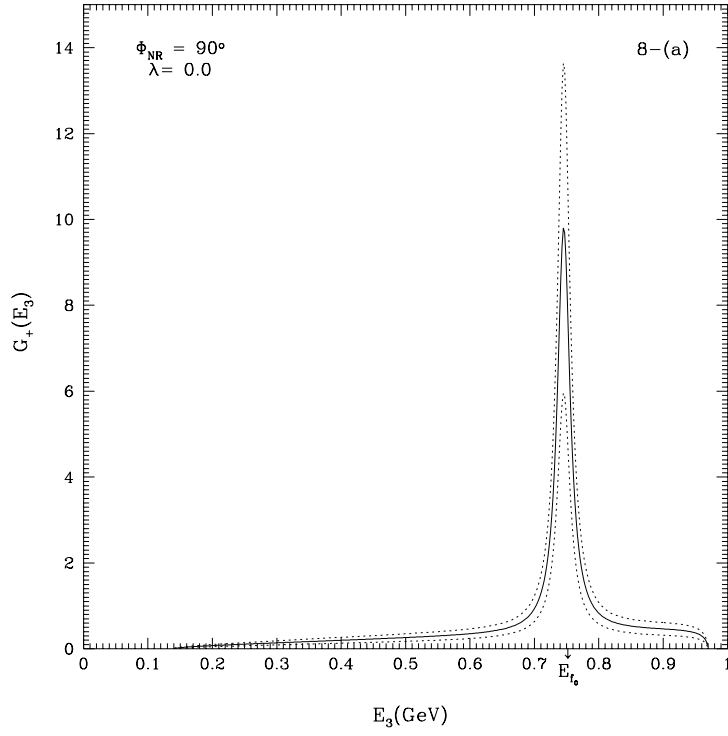


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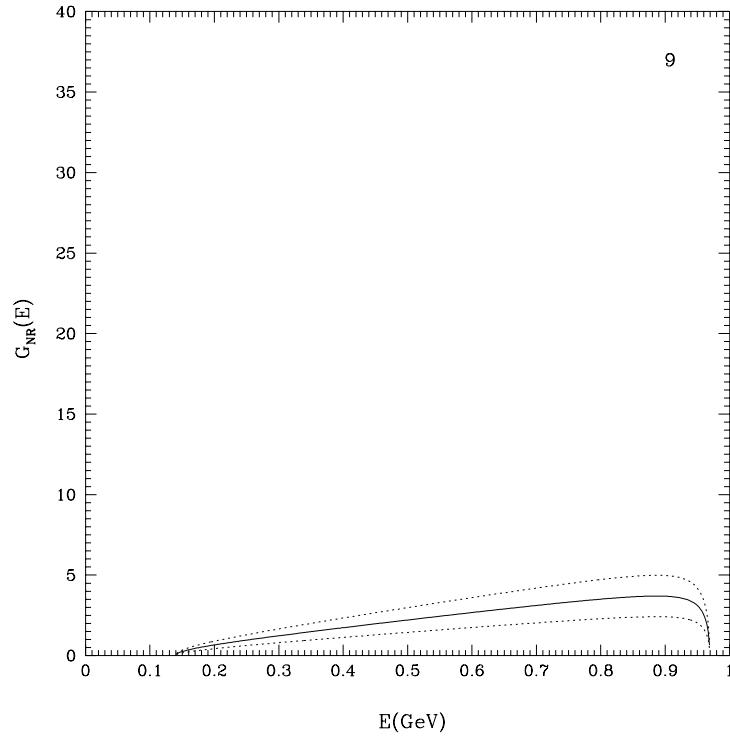


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